5.1. $y = f_a(x) = a \cdot \ln(a + x)$

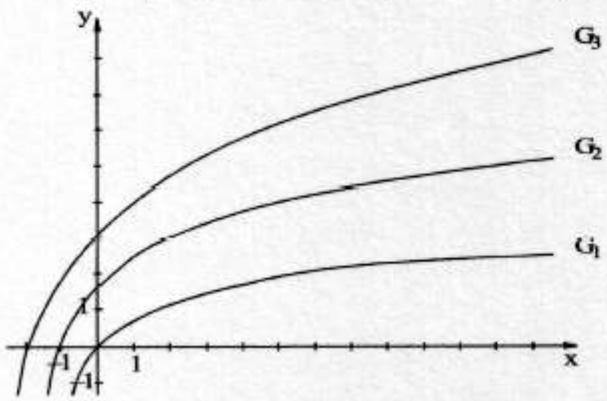
Definitionsbereich: a + x > 0 → x ∈ R, x > -a, a ∈ R, a > 0

$f_a(0) = a \cdot \ln a$ → $P_y(0 \mid a \cdot \ln a)$

$0 = a \cdot \ln(a + x)$ → $0 = \ln(a + x)$ → 1 = a + x → x = 1 - a → $P_x(1 - a \mid 0)$

5.2.

x	–2,5	–2	–1,5	–1	–0,5	0	1	2	3	4	5
$f_1(x) = 1 \cdot \ln(1 + x)$	–	–	–	–	–0,69	0	0,69	1,09	1,37	1,61	1,79
$f_2(x) = 2 \cdot \ln(2 + x)$	–	–	–1,39	0	0,81	1,39	2,19	2,77	3,22	3,58	3,89
$f_3(x) = 3 \cdot \ln(3 + x)$	–2,07	0	1,22	2,08	2,75	3,29	4,16	4,83	5,37	5,84	6,24



5.3. $P_x(1 - a \mid 0)$

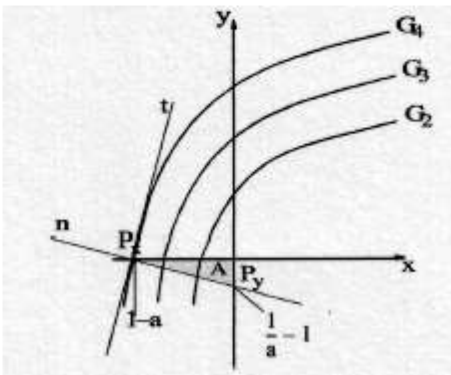
Normalengleichung: y = mx + n mit $m = -\frac{1}{f_a'(1 - a)}$

$f_a'(x) = a \cdot \frac{1}{a + x}$ → $f_a'(1 - a) = a \cdot \frac{1}{a + (1 - a)} = a$ → $m = -\frac{1}{a}$

→ $y = -\frac{1}{a}x + n$ → $0 = -\frac{1}{a}(1 - a) + n$ → $n = \frac{1}{a} - 1$

gesuchte Normalengleichung: $y = -\frac{1}{a}x + \frac{1}{a} - 1$

Dreieck laut Skizze:

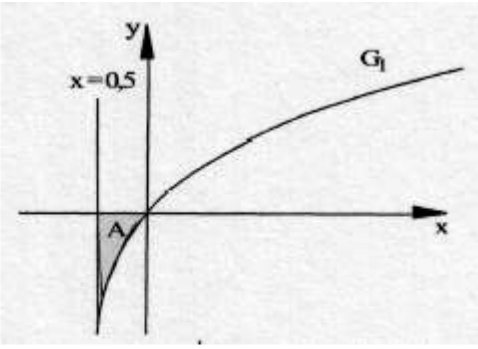


$A = \frac{1}{2}\left|\overline{P_xO}\right| \cdot \left|\overline{P_yO} = 1{,}125\right|$ mit $\left|\overline{P_xO}\right| = |1 - a| = a - 1$ und $\left|\overline{P_yO}\right| = \left|\frac{1}{a} - 1\right| = 1 - \frac{1}{a}$ wegen a > 1

also $\frac{1}{2}(a - 1)\left(1 - \frac{1}{a}\right) = 1{,}125$ → $a - 1 - 1 + \frac{1}{a} = 2{,}25$ → $a - 2 + \frac{1}{a} = \frac{9}{4}$ → $a^2 - 2a + 1 = \frac{9}{4}a$

bzw. $a^2 - \frac{17}{4}a + 1 = 0$ → a = 4 (Die zweite Lösung $a = \frac{1}{4}$ entfällt wegen a > 1!)

5.4.



Flächeninhalt mit $y = g(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + x$

$$A_* = \left| \int_{-0,5}^{0} \left(\frac{1}{3}x^3 - \frac{1}{2}x^2 + x \right) dx \right| = \left| \left[\frac{1}{12}x^4 - \frac{1}{6}x^3 + \frac{1}{2}x^2 \right]_{-0,5}^{0} \right| \approx 0,1510 FE$$

Flächeninhalt mit $y = f_1(x) = \ln(1+x)$

$$A = \left| \int_{-0,5}^{0} \ln(1+x) dx \right| = \left| [(1+x)\ln(1+x) - (1+x)]_{-0,5}^{0} \right| \approx 0,1534 FE$$

$$Abweichung = \frac{|A - A_*|}{A} = \frac{0,0024}{0,1534} = 0,0156$$ also eine Abweichung von etwa 1,56 %